CS 33

Architecture and Optimization (2)

Limitations of Optimizing Compilers

- Operate under fundamental constraint
  - must not cause any change in program behavior
  - often prevents it from making optimizations that would only affect behavior under pathological conditions
- Most analysis is performed only within functions
  - whole-program analysis is too expensive in most cases
- Most analysis is based only on *static* information
  - compiler has difficulty anticipating run-time inputs

- When in doubt, the compiler must be conservative

Generally Useful Optimizations

- **Optimizations that you or the compiler should do regardless of processor / compiler**

- **Code Motion**
  - **reduce frequency with which computation performed, if it will always produce same result**
    - » **especially moving code out of loop**

```
void set_row(long *a, long *b,
   long i, long n){
    long j;
    for (j = 0; j < n; j++)
        a[n*i+j] = b[j];
}
```

→

```
    long j;
    long ni = n*i;
    for (j = 0; j < n; j++)
        a[ni+j] = b[j];
```

Reduction in Strength

- Replace costly operation with simpler one
- Shift, add instead of multiply or divide

```
16*x      -->   x << 4
```

  - utility is machine-dependent
  - depends on cost of multiply or divide instruction
    - » on some Intel processors, multiplies are 3x longer than adds
- Recognize sequence of products

```
for (i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    a[n*i + j] = b[j];
```

→

```
int ni = 0;
for (i = 0; i < n; i++) {
  for (j = 0; j < n; j++)
    a[ni + j] = b[j];
  ni += n;
}
```

Share Common Subexpressions

- **Reuse portions of expressions**
- **Compilers often not very sophisticated in exploiting arithmetic properties**

```
/* Sum neighbors of i,j */
up =    val[(i-1)*n + j  ];
down =  val[(i+1)*n + j  ];
left =  val[i*n     + j-1];
right = val[i*n     + j+1];
sum = up + down + left + right;
```

```
long inj = i*n + j;
up =    val[inj - n];
down =  val[inj + n];
left =  val[inj - 1];
right = val[inj + 1];
sum = up + down + left + right;
```

3 multiplications: i*n, (i–1)*n, (i+1)*n

1 multiplication: i*n

```
leaq   1(%rsi), %rax  # i+1
leaq   -1(%rsi), %r8  # i-1
imulq  %rcx, %rsi     # i*n
imulq  %rcx, %rax     # (i+1)*n
imulq  %rcx, %r8      # (i-1)*n
addq   %rdx, %rsi     # i*n+j
addq   %rdx, %rax     # (i+1)*n+j
addq   %rdx, %r8      # (i-1)*n+j
```

```
imulq    %rcx, %rsi   # i*n
addq     %rdx, %rsi   # i*n+j
movq     %rsi, %rax   # i*n+j
subq     %rcx, %rax   # i*n+j-n
leaq     (%rsi,%rcx), %rcx # i*n+j+n
```

Quiz 1

The fastest means for evaluating

```
n*n + 2*n + 1
```

requires exactly:

a) 2 multiplies and 2 additions
b) three additions
c) one multiply and two additions
d) one multiply and one addition

Hint: remember high-school algebra

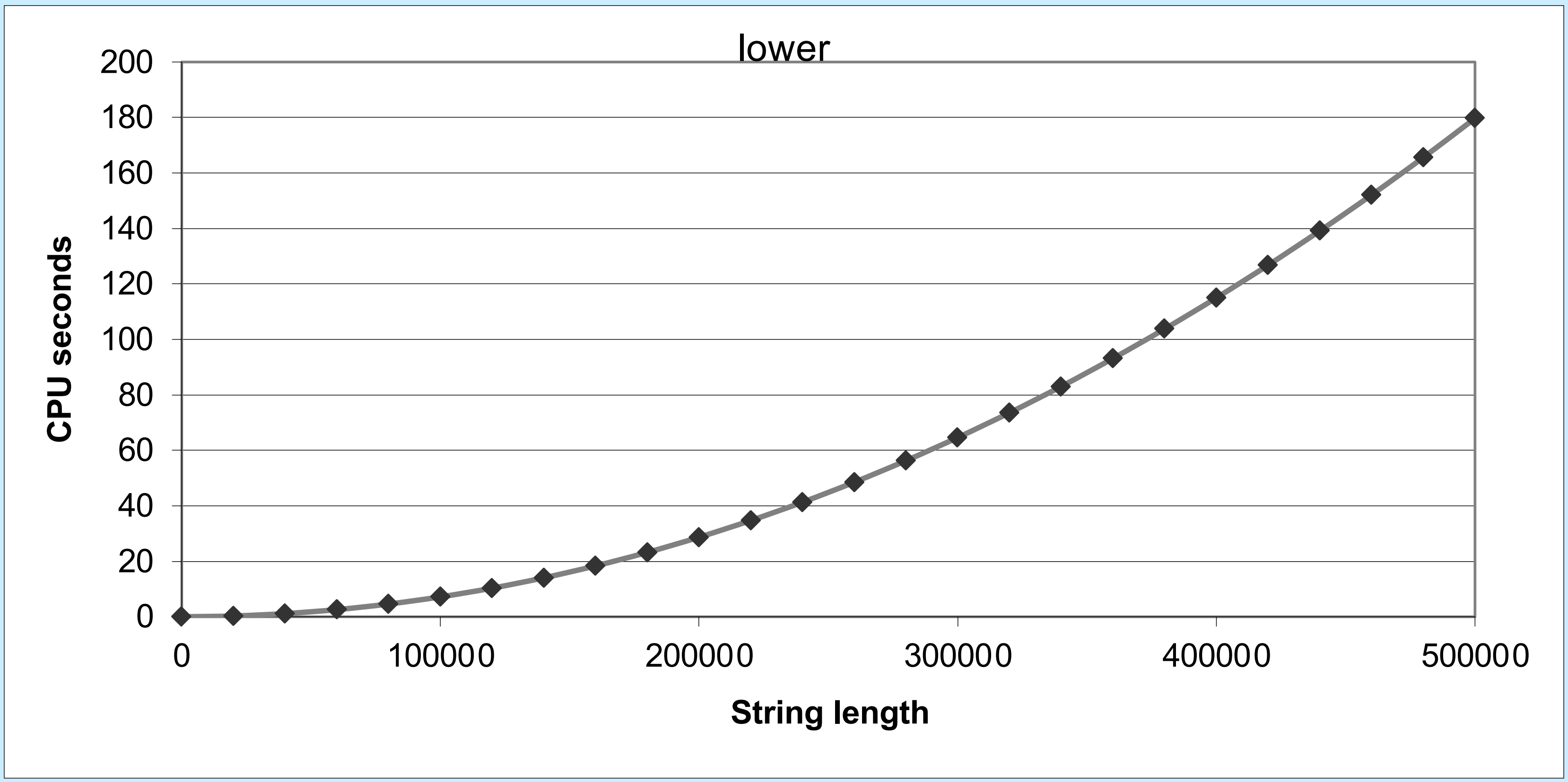
Optimization Blocker: Function Calls

- **Function to convert string to lower case**

```
void lower(char *s){
  int i;
  for (i = 0; i < strlen(s); i++)
    if (s[i] >= 'A' && s[i] <= 'Z')
      s[i] -= ('A' - 'a');
}
```

Lower Case Conversion Performance

- **Time quadruples when string length doubles**
- **Quadratic performance**



Convert Loop To Goto Form

```
void lower(char *s){
   int i = 0;
   if (i >= strlen(s))
     goto done;
 loop:
   if (s[i] >= 'A' && s[i] <= 'Z')
       s[i] -= ('A' - 'a');
   i++;
   if (i < strlen(s))
     goto loop;
 done:
}
```

- `strlen` **executed every iteration**

Strlen

```
size_t strlen(const char *s){
    size_t length = 0;
    while (*s != '\0') {
        s++;
        length++;
    }
    return length;
}
```

- **strlen performance**
  - only way to determine length of string is to scan its entire length, looking for null character
- **O(N) performance**
  - N calls to strlen
  - overall $O(N^2)$ performance

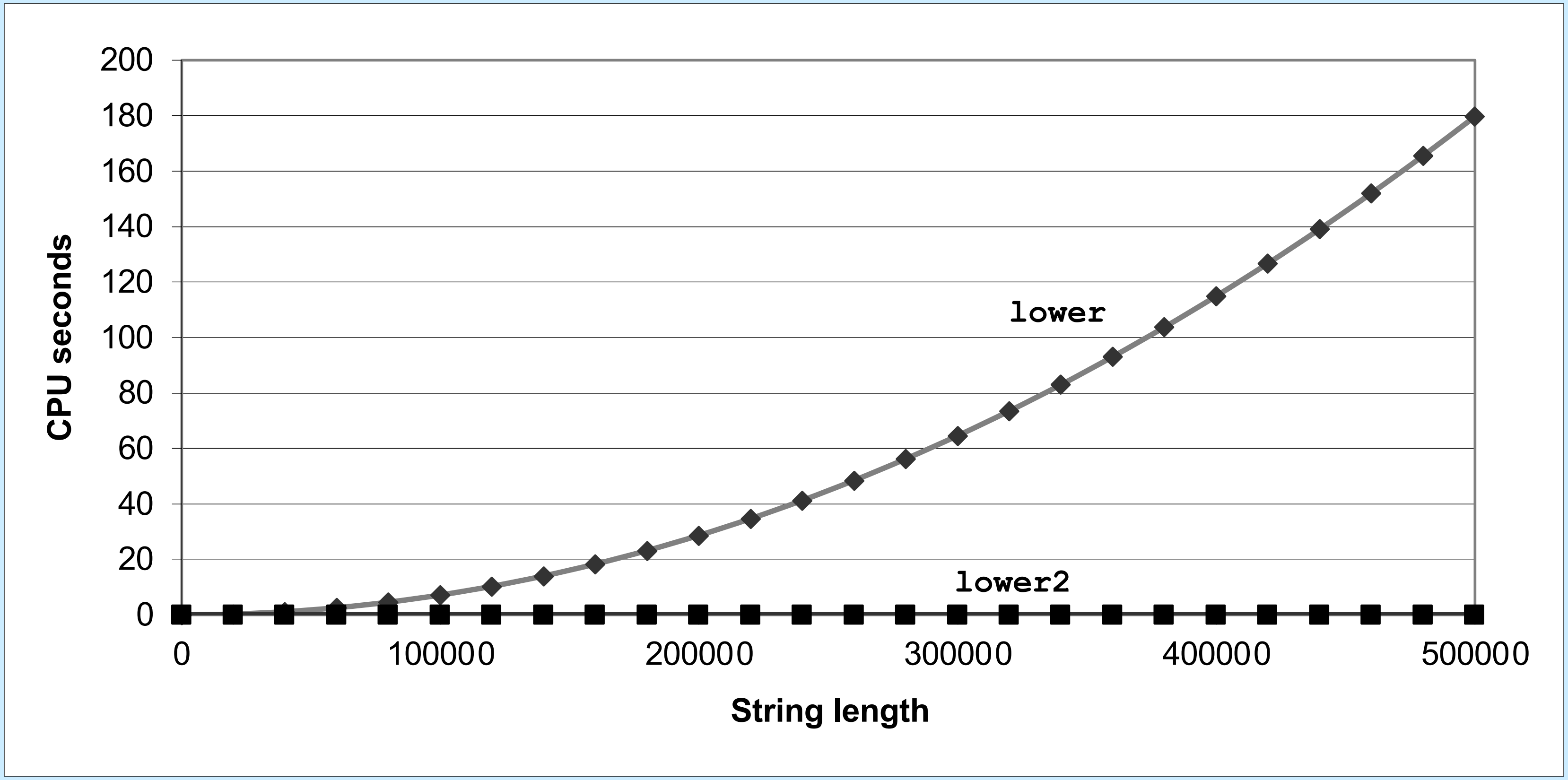
Improving Performance

```
void lower2(char *s){
  int i;
  int len = strlen(s);
  for (i = 0; i < len; i++)
    if (s[i] >= 'A' && s[i] <= 'Z')
      s[i] -= ('A' - 'a');
}
```

- **Move call to `strlen` outside of loop**
  - since result does not change from one iteration to another
  - form of code motion

Lower-Case Conversion Performance

- **Time doubles when string-length doubles**
  - **linear performance of lower2**



Optimization Blocker: Function Calls

- ***Why couldn't compiler move* `strlen` *out of inner loop?***
  - function may have side effects
    - » alters global state each time called
  - function may not return same value for given arguments
    - » depends on other parts of global state
    - » function `lower` could interact with `strlen`
- **Warning:**
  - compiler treats function call as a black box
  - weak optimizations near them
- **Remedy:**
  - do your own code motion

```
int lencnt = 0;
size_t strlen(const char *s){
    size_t length = 0;
    while (*s != '\0') {
       s++; length++;
    }
    lencnt += length;
    return length;
}
```

Memory Matters

```
/* Sum rows of n X n matrix a
   and store result in vector b  */
void sum_rows1(long n, long a[][n], long *b) {
    long i, j;
    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
            b[i] += a[i][j];
    }
}
```

```
# sum_rows1 inner loop
.L3:
   movq    (%r8,%rax,8), %rcx   # rcx = a[i][j]
   addq    %rcx, (%rdx)         # b[i] += rcx
   addq    $1, %rax             # j++
   cmpq    %rax, %rdi           # if i<n
   jne     .L3                  # goto .L3
```

- Code updates `b[i]` (in memory) on every iteration
- Why couldn't compiler optimize this away?

Memory Aliasing

```
/* Sum rows of n X n matrix a
   and store result in vector b  */
void sum_rows1(long n, long a[][n], long *b) {
    long i, j;
    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
            b[i] += a[i][j];
    }
}
```

```
int A[3][3] =
  {{ 0,   1,   2},
   { 4,   8,  16},
   {32,  64, 128}};

int *B = &A[1][0];

sum_rows1(3, A, B);
```

Value of B:

```
init:  [4, 8, 16]
```

```
i = 0: [3, 8, 16]
```

```
i = 1: [3, 22, 16]
```

```
i = 2: [3, 22, 224]
```

- **Code updates `b[i]` on every iteration**
- **Must consider possibility that these updates will affect program behavior**

Removing Aliasing

```
/* Sum rows of n X n matrix a
   and store result in vector b  */
void sum_rows1(long n, long a[][n], long *b) {
    long i, j;
    for (i = 0; i < n; i++) {
        long val = 0;
        for (j = 0; j < n; j++)
            val += a[i][j];
        b[i] = val;
    }
}
```

```
# sum_rows2 inner loop
.L4:
   addq    (%r8, %rax, 8), %rcx
   addq    $1, %rax
   cmpq    %rax, %rdi
   jne     .L4
```

- **No need to store intermediate results**

Optimization Blocker: Memory Aliasing

- Aliasing
  - two different memory references specify single location
  - easy to have happen in C
    - since allowed to do address arithmetic
    - direct access to storage structures
  - get in habit of introducing local variables
    - accumulating within loops
    - your way of telling compiler not to check for aliasing

C99 to the Rescue

- **New attribute**
  - **restrict**
    - » **applied to a pointer, tells the compiler that the object pointed to will be accessed only via this pointer**
    - » **compiler thus doesn't have to worry about aliasing**
    - » **but the programmer does ...**
    - » **syntax**

```
int *restrict pointer;
```

Pointers and Arrays

- **`long`** `a[][n]`
  - **a is a 2-D array of longs, the size of each row is n**
- **`long`** `(*c)[n]`
  - **c is a pointer to a 1-D array of size n**

- **a and c are of the same type**

Memory Matters, Fixed

```
/* Sum rows of n X n matrix a
   and store result in vector b  */
void sum_rows1(long n, long (*restrict a)[n], long *restrict b) {
    long i, j;
    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
            b[i] += a[i][j];
    }
}
```

```
# sum_rows1 inner loop
.L3:
   addq    (%rcx,%rax,8), %rdx
   addq    $1, %rax
   cmpq    %rax, %rdi
   jne     .L3
```

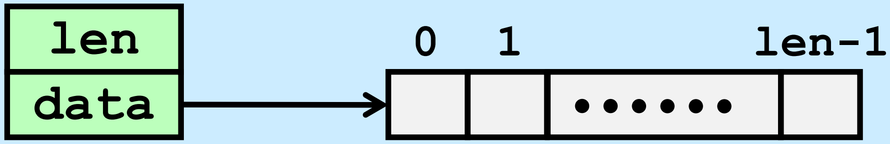
- **Code doesn't update `b[i]` on every iteration**

Exploiting Instruction-Level Parallelism

- **Need general understanding of modern processor design**
  - hardware can execute multiple instructions in parallel
- **Performance limited by data dependencies**
- **Simple transformations can have dramatic performance improvement**
  - compilers often cannot make these transformations
  - lack of associativity and distributivity in floating-point arithmetic

Benchmark Example: Datatype for Vectors

```
/* data structure for vectors */
typedef struct{
    int len;
    data_t *data;
} vec_t, *vec_ptr_t;
```




```
/* retrieve vector element and store at val */
int get_vec_element(vec_ptr_t v, int idx, data_t *val){
    if (idx < 0 || idx >= v->len)
        return 0;
    *val = v->data[idx];
    return 1;
}

/* return length of vector */
int vec_length(vec_ptr_t v) {
     return v->len;
}
```

Benchmark Computation

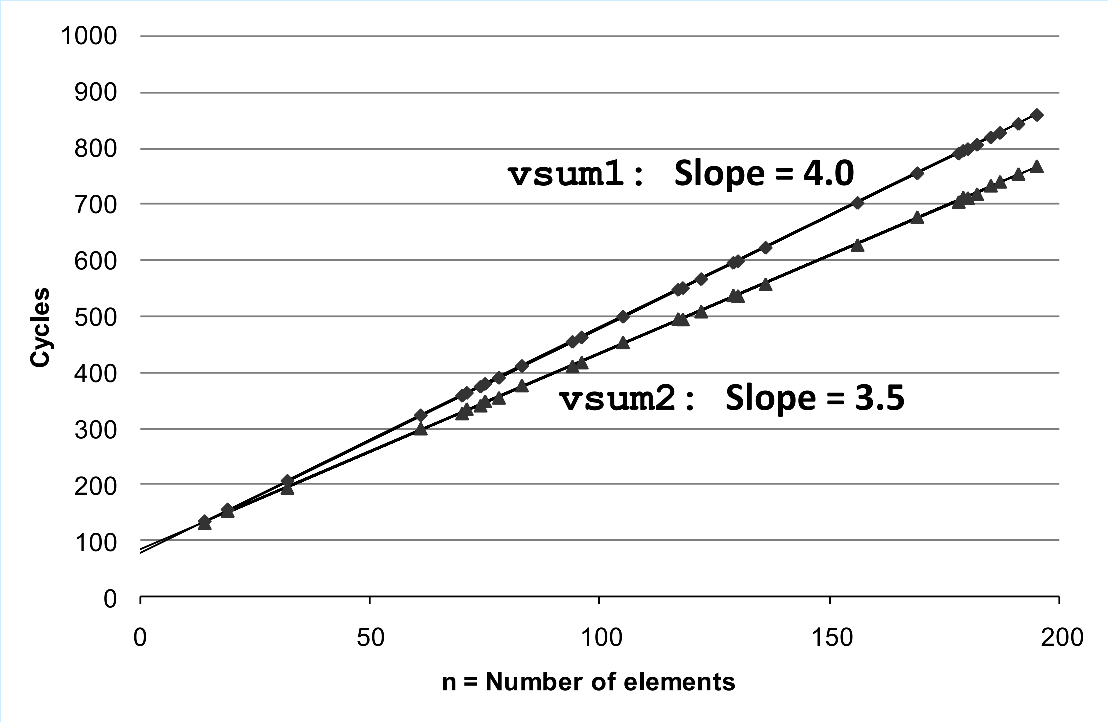
```
void combine1(vec_ptr_t v, data_t *dest){
    long int i;
    *dest = IDENT;
    for (i = 0; i < vec_length(v); i++) {
      data_t val;
      get_vec_element(v, i, &val);
      *dest = *dest OP val;
    }
}
```

Compute sum or product of vector elements

- **Data Types**
  - **use different declarations for `data_t`**
    - » `int`
    - » `float`
    - » `double`
- **Operations**
  - **use different definitions of `OP` and `IDENT`**
    - » `+, 0`
    - » `*, 1`

Cycles Per Element (CPE)

- **Convenient way to express performance of program that operates on vectors or lists**
- **Length = n**
- **T = CPE*n + Overhead**
  - **CPE is slope of line**



Benchmark Performance

```
void combine1(vec_ptr_t v, data_t *dest){
    long int i;
    *dest = IDENT;
    for (i = 0; i < vec_length(v); i++) {
      data_t val;
      get_vec_element(v, i, &val);
      *dest = *dest OP val;
    }
}
```

Compute sum or product of vector elements

Method	Integer		Double FP	
Operation	**Add**	**Mult**	**Add**	**Mult**
Combine1 unoptimized	**29.0**	**29.2**	**27.4**	**27.9**
Combine1 –O1	**12.0**	**12.0**	**12.0**	**13.0**

Move vec_length

```
void combine2(vec_ptr_t v, data_t *dest){
    long int i;
    long int length = vec_length(v);
    *dest = IDENT;
    for (i = 0; i < length; i++) {
       data_t val;
       get_vec_element(v, i, &val);
       *dest = *dest OP val;
    }
}
```

Method	Integer		Double FP	
Operation	Add	Mult	Add	Mult
Combine1 unoptimized	29.0	29.2	27.4	27.9
Combine1 –O1	12.0	12.0	12.0	13.0
Combine2	8.03	8.09	10.09	12.08

Eliminate Function Calls

```
void combine3(vec_ptr_t v, data_t *dest){
    long int i;
    long int length = vec_length(v);
    data_t *data = get_vec_start(v);
    *dest = IDENT;
    for (i = 0; i < length; i++) {
      *dest = *dest OP data[i];
    }
}
```

```
data_t *get_vec_start(
      vec_ptr v) {
  return v->data;
}
```

Method	Integer		Double FP	
Operation	Add	Mult	Add	Mult
Combine2	8.03	8.09	10.09	12.08
Combine3	6.01	8.01	10.01	12.02

Eliminate Unneeded Memory References

```
void combine4(vec_ptr_t v, data_t *dest){
  int i;
  int length = vec_length(v);
  data_t *d = get_vec_start(v);
  data_t t = IDENT;
  for (i = 0; i < length; i++)
    t = t OP d[i];
  *dest = t;
}
```

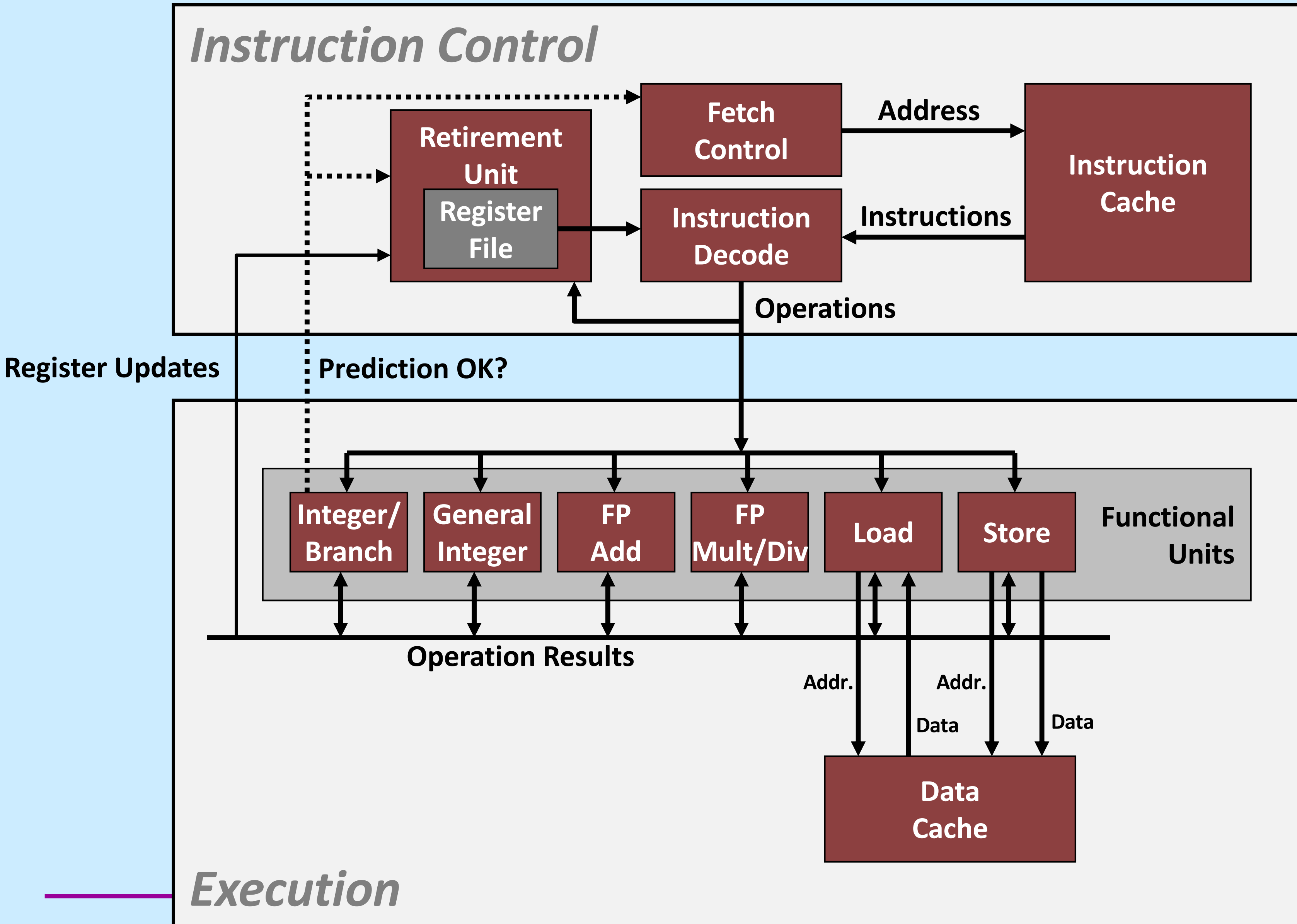
Method	Integer		Double FP	
Operation	Add	Mult	Add	Mult
Combine1 –O1	12.0	12.0	12.0	13.0
Combine4	2.0	3.0	3.0	5.0

Not a Quiz

Combine4 is pretty fast; we've done all the "obvious" optimizations. How much faster will we be able to make it? (Hint: it involves taking advantage of pipelining and multiple functional units on the chip.)

a) 1× (it's already as fast as possible)
b) 2× – 4×
c) 16× – 64×
d) 128× – ∞×

Modern CPU Design



Multiple Operations per Instruction

- **addq %rax, %rdx**
  - a single operation
- **addq %rax, 8(%rdx)**
  - three operations
    - » load value from memory
    - » add to it the contents of %rax
    - » store result in memory

Instruction-Level Parallelism

- `addq 8(%rax), %rax`
  `addq %rbx, %rdx`
  – can be executed simultaneously: completely independent
- `addq 8(%rax), %rbx`
  `addq %rbx, %rdx`
  – can also be executed simultaneously, but some coordination is required

Out-of-Order Execution

```
• movss    (%rbp), %xmm0
  mulss    (%rax, %rdx, 4), %xmm0
  movss    %xmm0, (%rbp)
  addq     %r8, %r9
  imulq    %rcx, %r12
  addq     $1, %rdx
```

these can be executed without waiting for the first three to finish (applies to the last three instructions)

Speculative Execution

```
80489f3:    movl    $0x1,%ecx
80489f8:    xorq    %rdx,%rdx
80489fa:    cmpq    %rsi,%rdx
80489fc:    jnl     8048a25
80489fe:    movl    %esi,%edi
8048a00:    imull   (%rax,%rdx,4),%ecx
```

perhaps execute these instructions (the last two: 80489fe and 8048a00)

Haswell CPU

- **Functional Units**
  1) Integer arithmetic, floating-point multiplication, integer and floating-point division, branches
  2) Integer arithmetic, floating-point addition, integer and floating-point multiplication
  3) Load, address computation
  4) Load, address computation
  5) Store
  6) Integer arithmetic
  7) Integer arithmetic, branches
  8) Store, address computation

Haswell CPU

- **Instruction characteristics**

Instruction	*Latency*	*Cycles/Issue*	*Capacity*
Integer Add	1	1	4
Integer Multiply	3	1	1
Integer/Long Divide	3-30	3-30	1
Single/Double FP Add	3	1	1
Single/Double FP Multiply	5	1	2
Single/Double FP Divide	3-15	3-15	1
Load	4	1	2
Store	-	1	2

Haswell CPU Performance Bounds

	Integer		Floating Point	
	+	*	+	*
Latency	1.00	3.00	3.00	5.00
Throughput	4.00	1.00	1.00	2.00

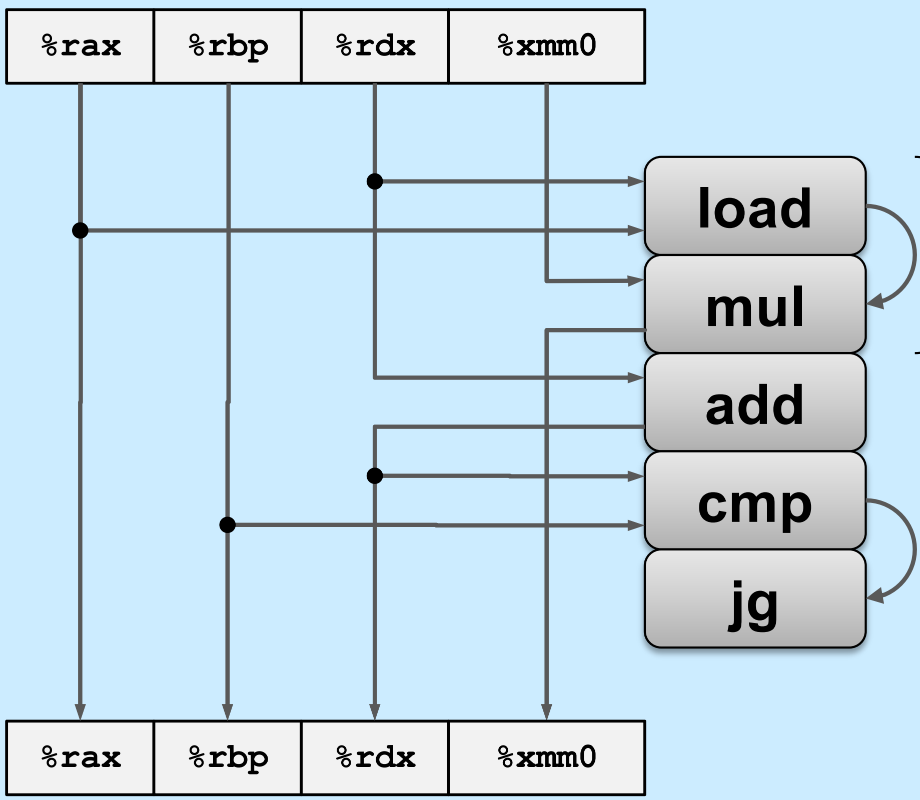
x86-64 Compilation of Combine4

- Inner loop (case: SP floating-point multiply)

```
.L519:                                # Loop:
  mullss (%rax,%rdx,4), %xmm0 # t = t * d[i]
  addq  $1, %rdx                    # i++
  cmpq  %rdx, %rbp                  # Compare length:i
  jg    .L519                       # If >, goto Loop
```

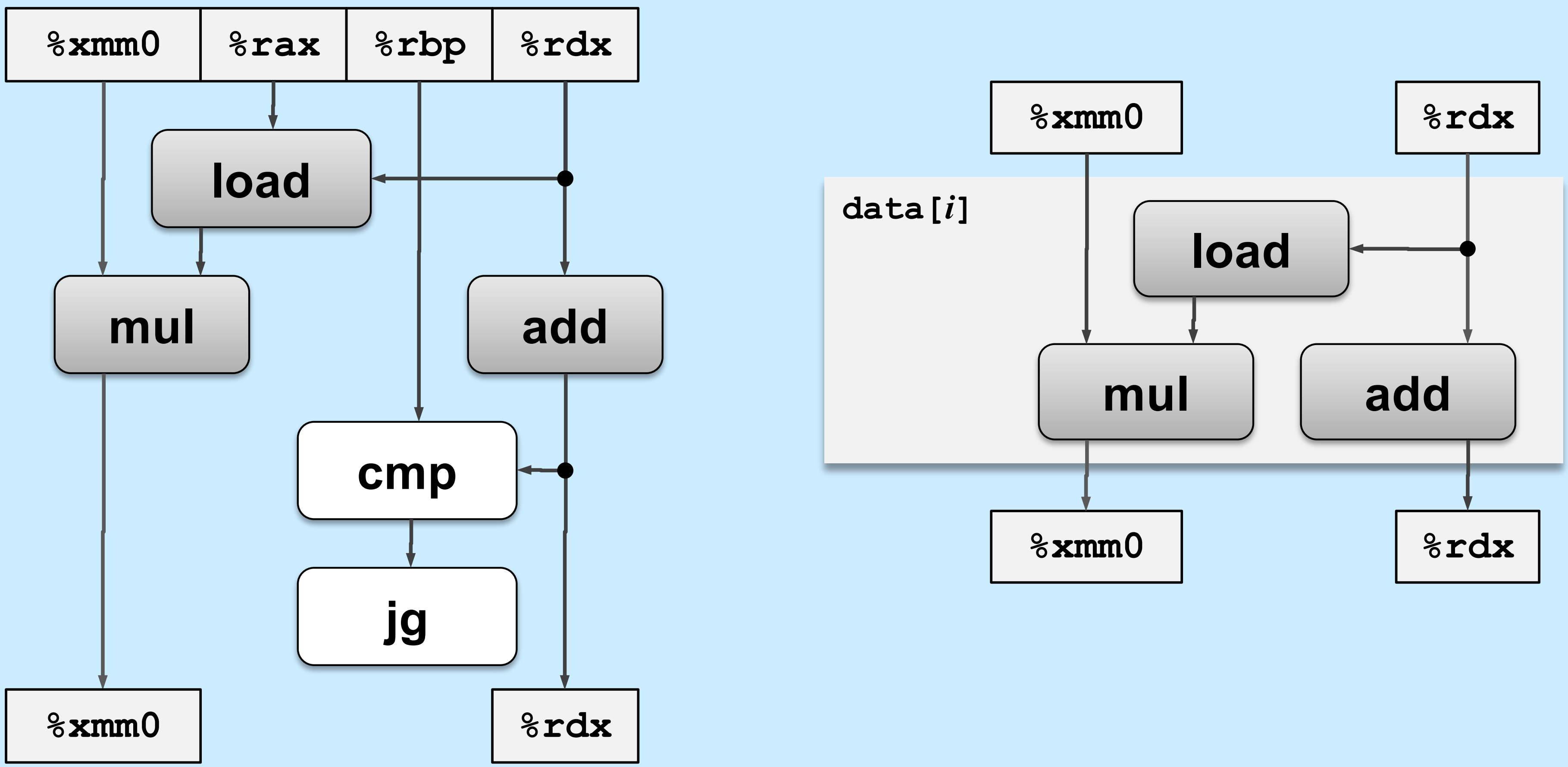
Method	Integer		Double FP	
Operation	Add	Mult	Add	Mult
Combine4	1.27	3.00	3.00	5.00
Latency bound	1.00	3.00	3.00	5.0
Throughput bound	0.25	1.00	1.00	0.50

Inner Loop

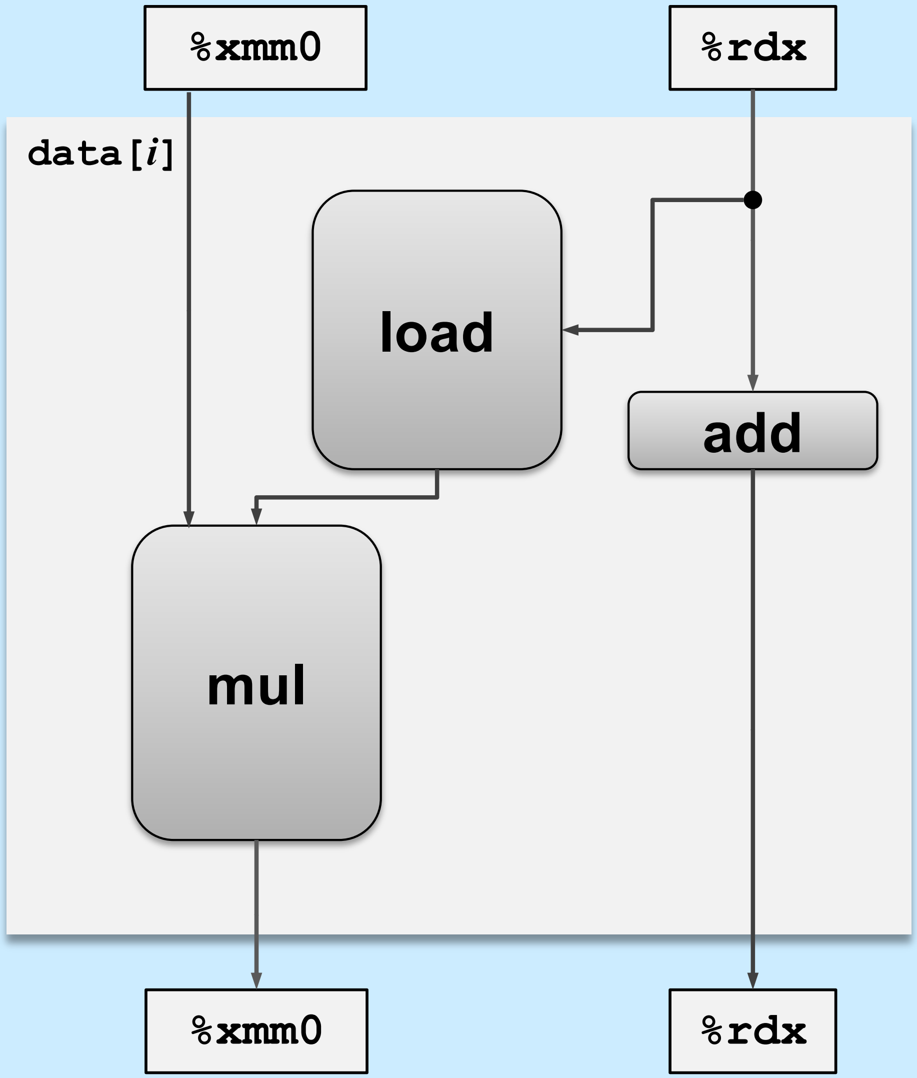



```
mulss (%rax,%rdx,4), %xmm0
addq $1,%rdx
cmpq %rdx,%rbp
jg loop
```

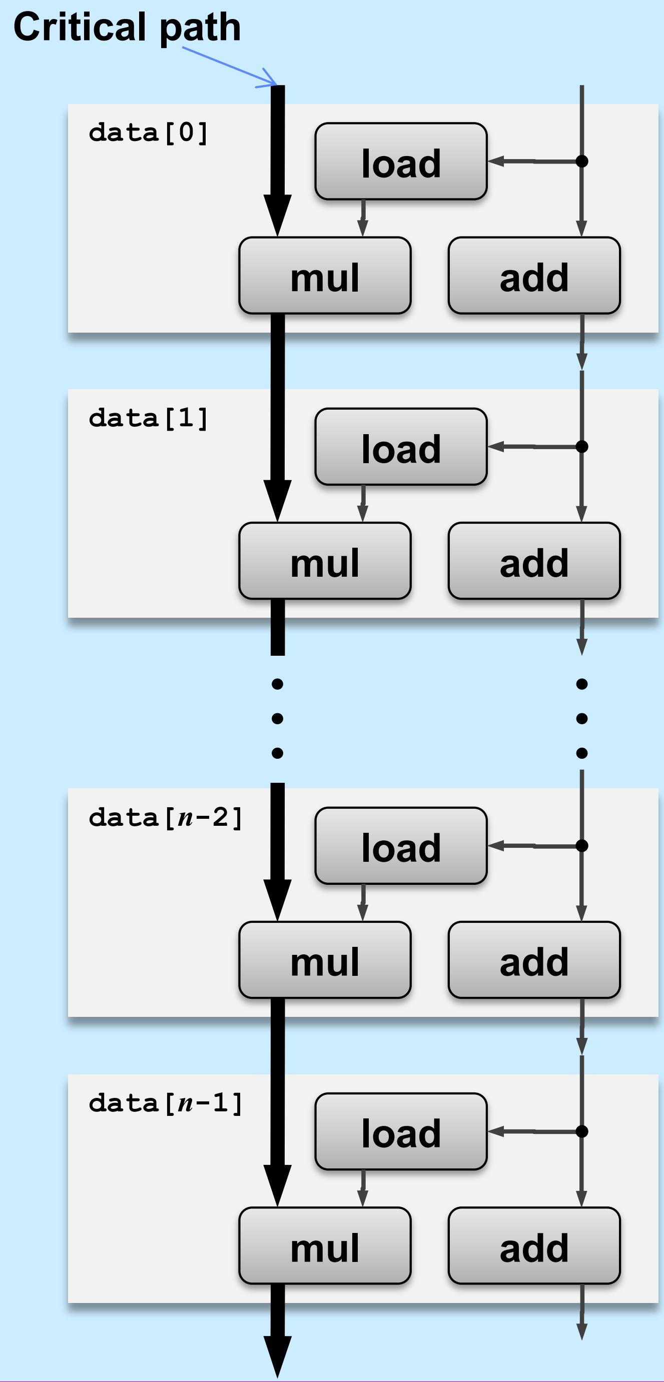
Data-Flow Graphs of Inner Loop



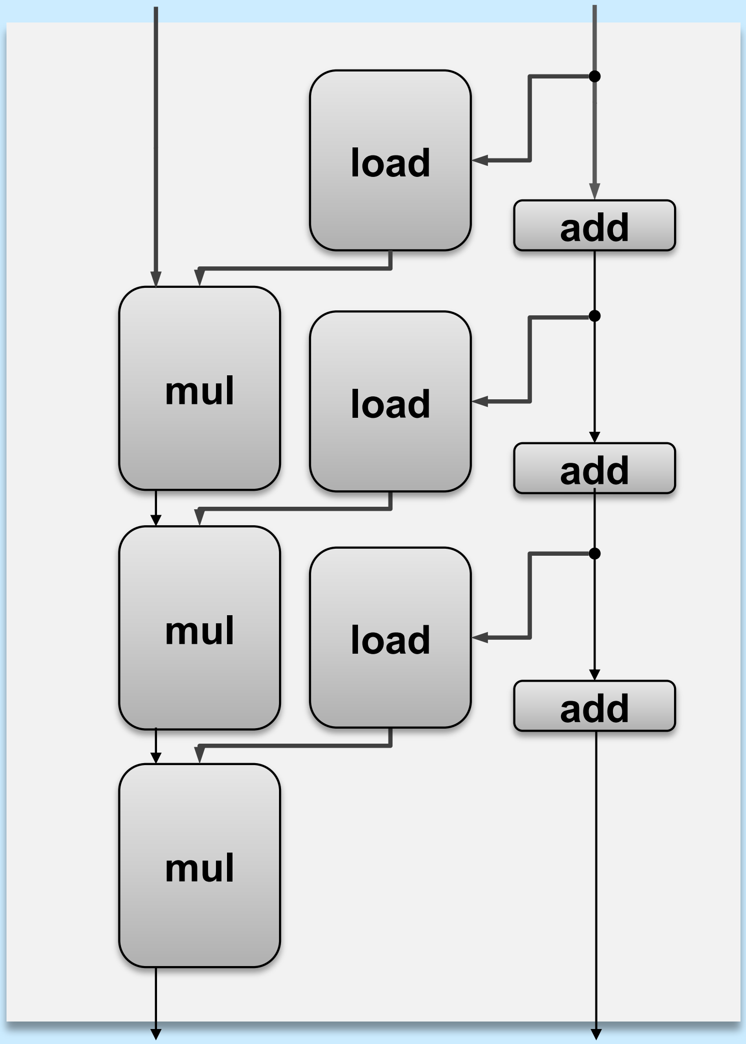
Relative Execution Times



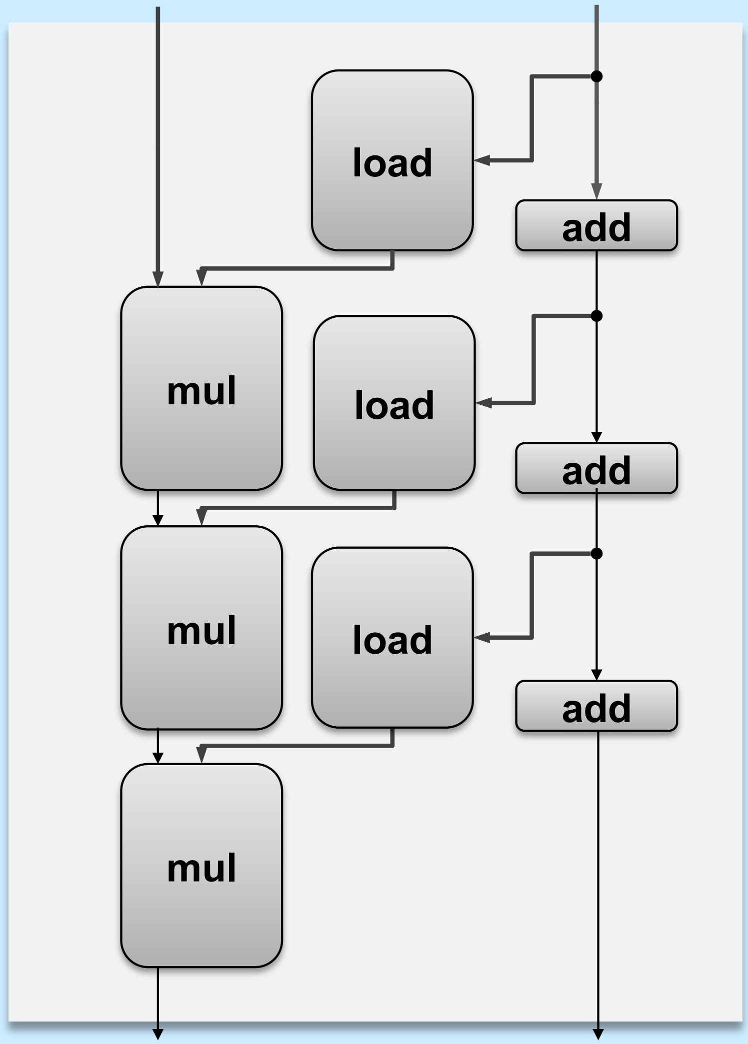
Data Flow Over Multiple Iterations



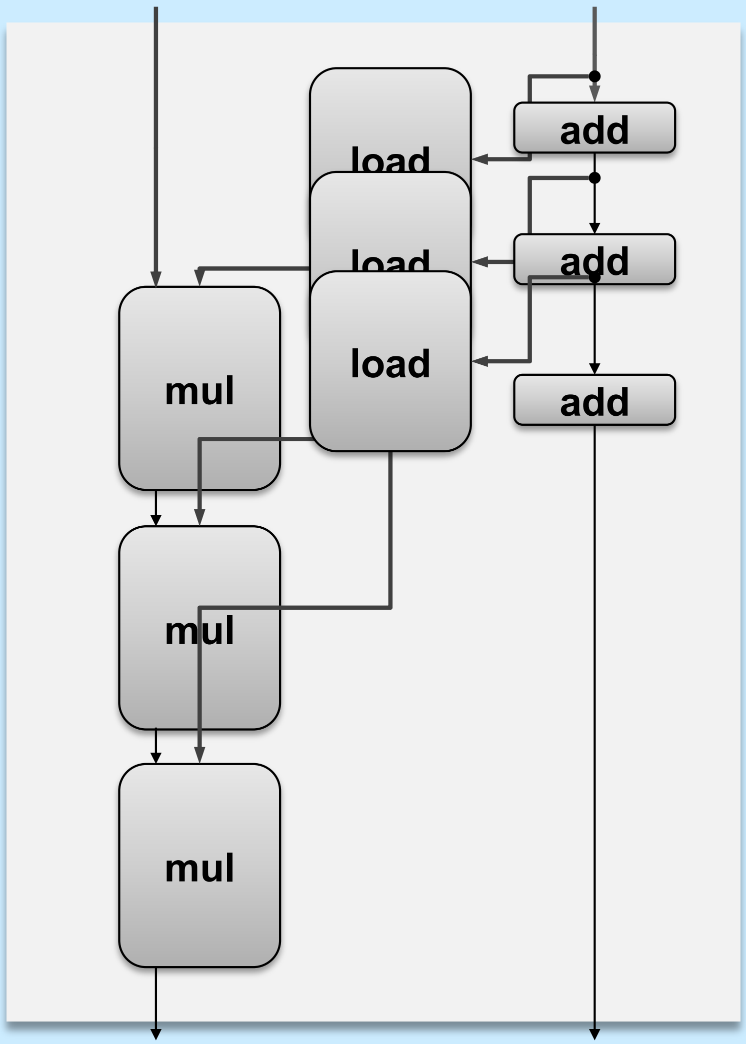
Pipelined Data-Flow Over Multiple Iterations



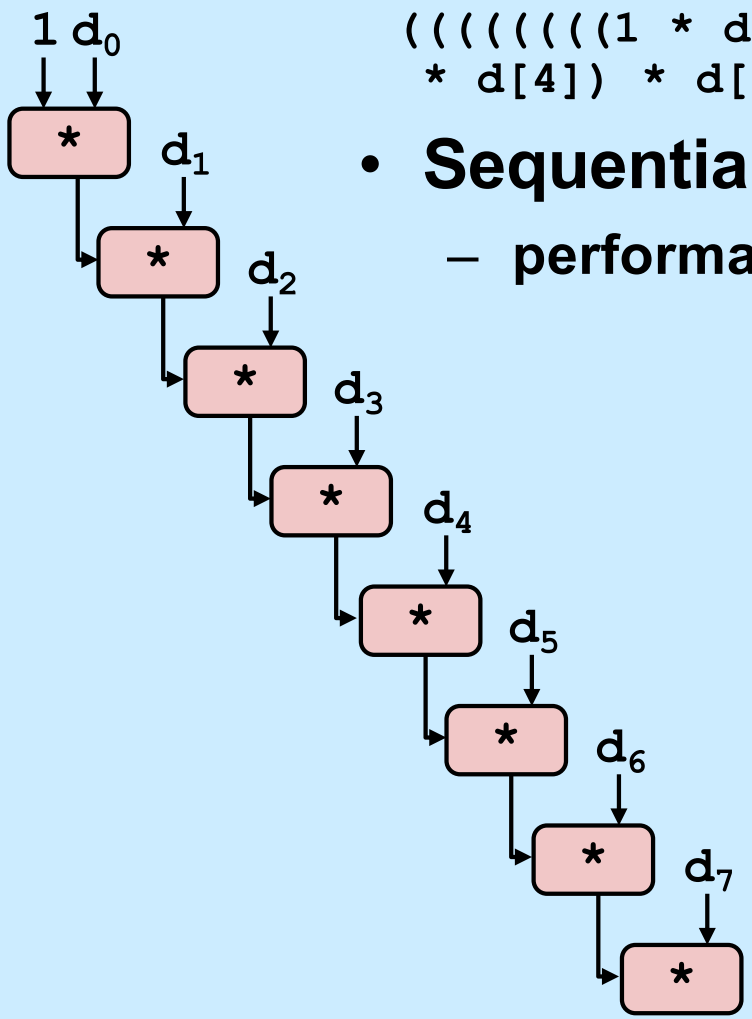
Pipelined Data-Flow Over Multiple Iterations



Pipelined Data-Flow Over Multiple Iterations



Combine4 = Serial Computation (OP = *)



- **Computation (length=8)**

```
((((((((1 * d[0]) * d[1]) * d[2]) * d[3])
 * d[4]) * d[5]) * d[6]) * d[7])
```

- **Sequential dependence**
  - **performance: determined by latency of OP**

Loop Unrolling

```
void unroll2x(vec_ptr_t v, data_t *dest)
{
    int length = vec_length(v);
    int limit = length-1;
    data_t *d = get_vec_start(v);
    data_t x = IDENT;
    int i;
    /* Combine 2 elements at a time */
    for (i = 0; i < limit; i+=2) {
        x = (x OP d[i]) OP d[i+1];
    }
    /* Finish any remaining elements */
    for (; i < length; i++) {
        x = x OP d[i];
    }
    *dest = x;
}
```

- **Perform 2x more useful work per iteration**

Effect of Loop Unrolling

Method	Integer		Double FP	
Operation	Add	Mult	Add	Mult
Combine4	1.27	3.00	3.00	5.00
Unroll 2x	1.01	3.00	3.00	5.00
Latency bound	1.0	3.0	3.0	5.0
Throughput bound	0.25	1.0	1.0	0.5

- **Helps integer add**
  - reduces loop overhead
- **Others don't improve. *Why?***
  - still sequential dependency

```
x = (x OP d[i]) OP d[i+1];
```

Loop Unrolling with Reassociation

```
void unroll2xra(vec_ptr_t v, data_t *dest)
{
    int length = vec_length(v);
    int limit = length-1;
    data_t *d = get_vec_start(v);
    data_t x = IDENT;
    int i;
    /* Combine 2 elements at a time */
    for (i = 0; i < limit; i+=2) {
        x = x OP (d[i] OP d[i+1]);
    }
    /* Finish any remaining elements */
    for (; i < length; i++) {
        x = x OP d[i];
    }
    *dest = x;
}
```

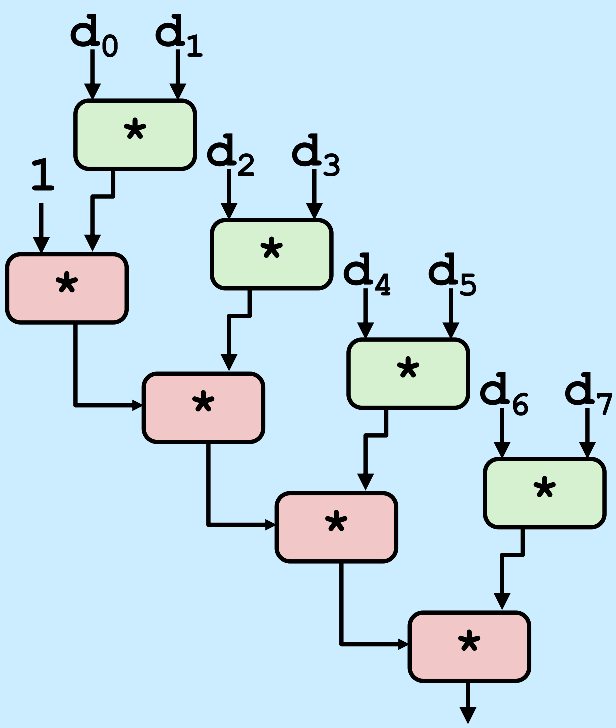
Compare to before

```
x = (x OP d[i]) OP d[i+1];
```

- **Can this change the result of the computation?**
- **Yes, for FP. *Why?***

Reassociated Computation

```
x = x OP (d[i] OP d[i+1]);
```



- **What changed:**
  - ops in the next iteration can be started early (no dependency)
- **Overall Performance**
  - N elements, D cycles latency/op
  - should be (N/2+1)*D cycles: **CPE = D/2**
  - measured CPE slightly worse for integer addition (there are other things going on)

Effect of Reassociation

Method	Integer		Double FP	
Operation	Add	Mult	Add	Mult
Combine4	1.27	3.00	3.00	5.00
Unroll 2x	1.01	3.00	3.00	5.00
Unroll 2x, reassociate	1.01	1.51	1.51	2.51
Latency bound	1.0	3.0	3.0	5.0
Throughput bound	.25	1.0	1.0	.5

- Nearly 2x speedup for int *, FP +, FP *
  - reason: breaks sequential dependency

```
x = x OP (d[i] OP d[i+1]);
```

Loop Unrolling with Separate Accumulators

```
void unroll2xp2x(vec_ptr_t v, data_t *dest)
{
    int length = vec_length(v);
    int limit = length-1;
    data_t *d = get_vec_start(v);
    data_t x0 = IDENT;
    data_t x1 = IDENT;
    int i;
    /* Combine 2 elements at a time */
    for (i = 0; i < limit; i+=2) {
       x0 = x0 OP d[i];
       x1 = x1 OP d[i+1];
    }
    /* Finish any remaining elements */
    for (; i < length; i++) {
        x0 = x0 OP d[i];
    }
    *dest = x0 OP x1;
}
```

- **Different form of reassociation**

Effect of Separate Accumulators

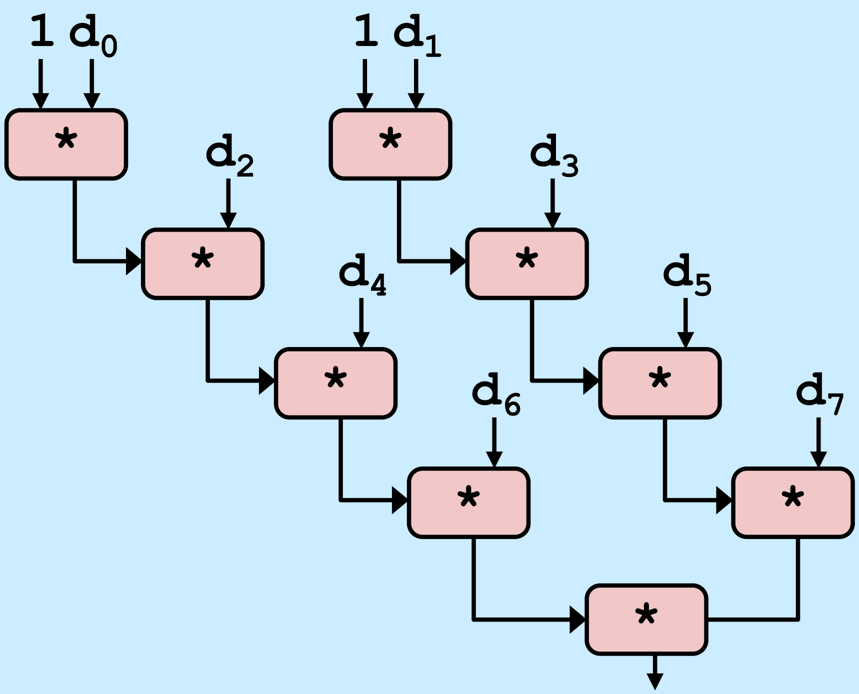
Method	Integer		Double FP	
Operation	Add	Mult	Add	Mult
Combine4	1.27	3.00	3.00	5.00
Unroll 2x	1.01	3.00	3.00	5.00
Unroll 2x, reassociate	1.01	1.51	1.51	2.01
Unroll 2x parallel 2x	.81	1.51	1.51	2.51
Latency bound	1.0	3.0	3.0	5.0
Throughput bound	.25	1.0	1.0	.5

- 2x speedup (over unroll 2x) for int *, FP +, FP *
  - breaks sequential dependency in a “cleaner,” more obvious way

```
x0 = x0 OP d[i];
x1 = x1 OP d[i+1];
```

Separate Accumulators

```
x0 = x0 OP d[i];
x1 = x1 OP d[i+1];
```



- **What changed:**
  - two independent “streams” of operations
- **Overall Performance**
  - N elements, D cycles latency/op
  - should be (N/2+1)*D cycles: **CPE = D/2**
  - Integer addition improved, but not yet at predicted value

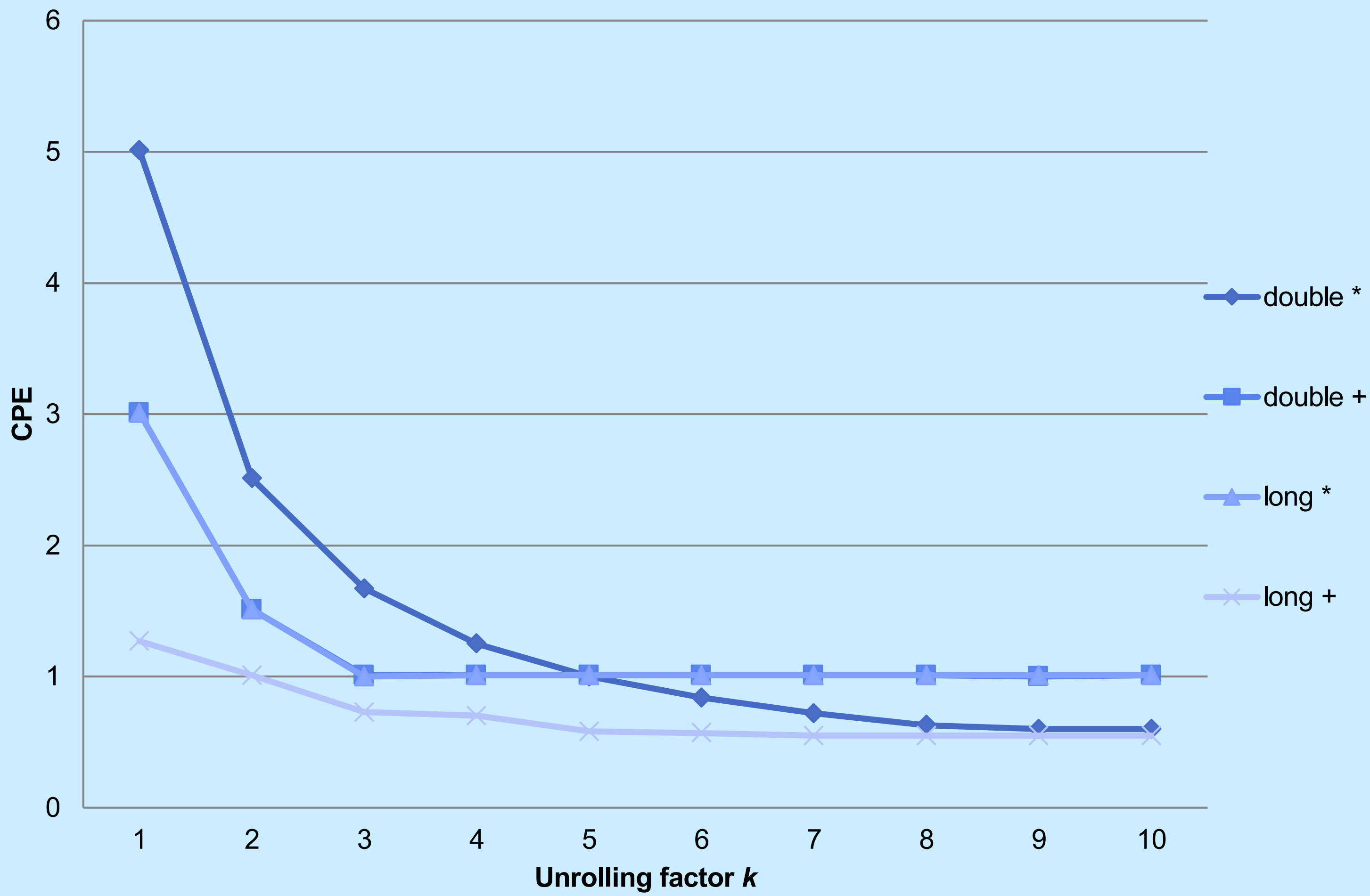
What Now?

Quiz 2

We’re making progress. With two accumulators we get a two-fold speedup. With three accumulators, we can get a three-fold speedup. How much better performance can we expect if we add even more accumulators?

a) It keeps on getting better as we add more and more accumulators
b) It’s limited by the latency bound
c) It’s limited by the throughput bound
d) It’s limited by something else

Performance



- K-way loop unrolling with K accumulators
  - limited by number and throughput of functional units

Achievable Performance

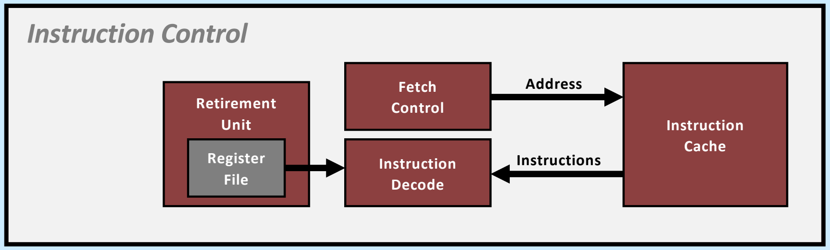
Method	Integer		Double FP	
Operation	Add	Mult	Add	Mult
Combine4	1.27	3.0	3.0	5.0
Achievable scalar	.52	1.01	1.01	.54
Latency bound	1.00	3.00	3.00	5.00
Throughput bound	.25	1.00	1.00	.5

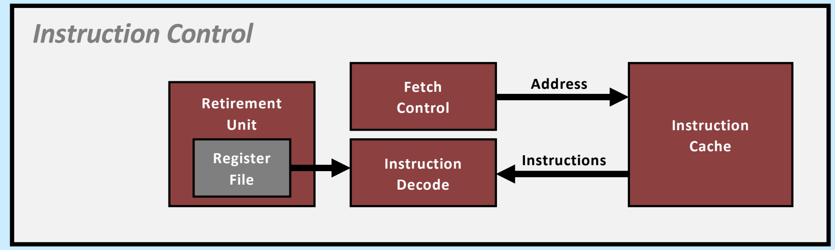
Using Vector Instructions

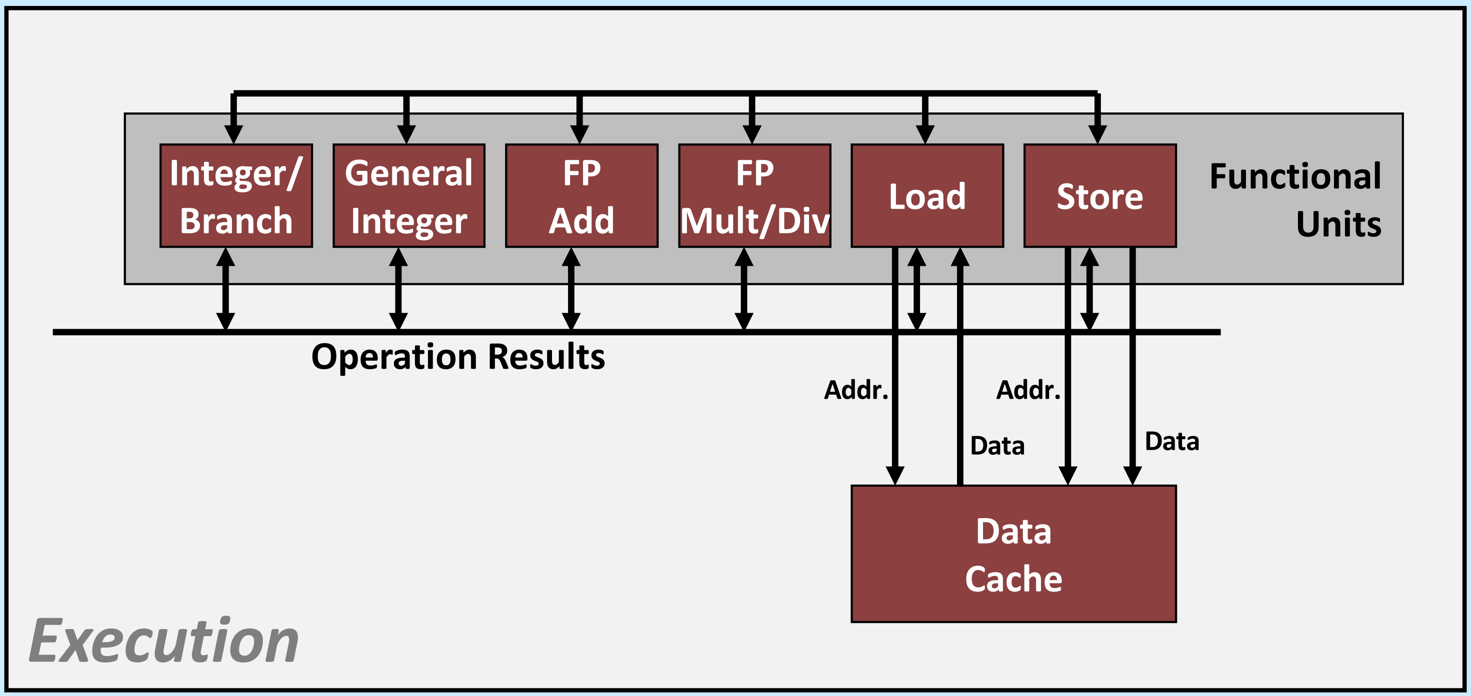
Method	Integer		Double FP	
Operation	Add	Mult	Add	Mult
Combine4	1.27	3.0	3.0	5.0
Achievable Scalar	.52	1.01	1.01	.54
Latency bound	1.00	3.00	3.00	5.00
Throughput bound	.25	1.00	1.00	.5
Achievable Vector	.05	.24	.25	.16
Vector throughput bound	.06	.12	.25	.12

- **Make use of SSE Instructions**
  - parallel operations on multiple data elements

Hyper Threading







Multiple Cores